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Searching for dark matter constituents with many solar masses

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Searches for dark matter (DM) constituents are presently mainly focused on axions and weakly interacting massive particle (WIMPs) despite the fact that far higher mass constituents are viable. We discuss and dispute whether axions exist and those arguments for WIMPs which arise from weak scale supersymmetry. We focus on the highest possible masses and argue that, since if they constitute all DM, they cannot be baryonic, they must uniquely be primordial black holes. Observational constraints require them to be of intermediate masses mostly between ten and a hundred thousand solar masses. Known search strategies for such PIMBHs include wide binaries, cosmic microwave background (CMB) distortion and, most promisingly, extended microlensing experiments.

Keywords: Dark matter; black holes; microlensing; wide binaries; CMB distortion.

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1. Introduction

Astronomical observations have led to a consensus that the energy make-up of the visible universe is approximately 70% dark energy, 25% dark matter (DM) and only 5% normal matter. The dark energy remains mainly mysterious; the DM problem will be addressed in the present paper; and the normal matter has a successful theory applicable up to at least a few hundred GeV in the form of the Standard Model.[a]

General discussions of the history and experiments for DM are in Refs. 2–4. A recent popular book[5] is strong on the panoply of unsuccessful weakly interacting massive particle (WIMP) searches.

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[a] More precise numbers from the Planck Collaboration[1] are 68.4% dark energy, 26.7% DM and 4.9% normal matter.

In this paper, we shall make the unjustified assumption that there is only one species of DM. Because the luminous matter is far richer than this, with its varied menu of the three families of quarks and leptons, the gauge bosons and the Brout–Englert–Higgs (BEH) boson, there is no sharp reason why the DM should be so different and simpler. One practical reason to make such an assumption is to simplify the research. A better reason is that it is likely, in our opinion, to be correct. If it is not so, then our conclusions may apply only to one component, perhaps the dominant component of the DM.

The present ignorance of the DM sector is put into perspective by looking at the uncertainty in the values of the constituent mass previously considered. The lightest such candidate is the invisible axion with $M = 1$ μeV. One very massive such candidate is the intermediate mass black hole (IMBH) with $M = 100{,}000\,M_\odot$ which is a staggering 77 orders of magnitude larger. Our aim is to reduce this uncertainty.

A result of the present analysis will be that the number of orders of magnitude uncertainty in the DM constituent mass can be reduced to four. We shall conclude, after extensive discussion, that the most viable candidate for the constituent which dominates DM is the IMBH with mass in the range

$$10\,M_\odot < M_{\mathrm{IMBH}} < 100{,}000\,M_\odot\,. \tag{1}$$

Less experimental effort is being invested in searching for IMBHs than for WIMPs. WIMP searches include terrestrial direct detection, astronomical indirect detection and production of WIMPs at the LHC.

One reason for the neglect of IMBHs may be that the literature is confusing including one study[6] which claimed entirely to rule out Eq. (1). We shall attempt to clarify the situation which actually still permits the whole range in Eq. (1). The present paper is, in part, an attempt to redress the imbalance between the few experimental efforts to search for IMBHs compared to the extensive WIMP searches.

One possible reason for previously overlooking our solution to the DM problem is that it had been assumed that all black holes arise only from gravitational collapse of baryonic objects, either normal stars or superheavy early stars.

Some of this paper is on review of old topics but it is with an original viewpoint to try to identify what is the most likely constituent of the DM, more by logic and partial elimination of popular possibilities than by new calculations.

The plan of this paper is that in Sec. 2, we discuss both axions and WIMPS, then in Sec. 3, examples of massive compact halo objects (MACHOs) including the IBMHs are discussed. Three experimental methods to search for IMBHs are discussed in Sec. 4 (wide binaries), Sec. 5 (distortion of the cosmic microwave background (CMB)) and Sec. 6 (microlensing). Finally Sec. 7 is devoted to further discussion.

2. Axions and WIMPs

2.1. *Axions*

It is worth reviewing briefly the history of the axion particle now believed, if it exists, to lie in the mass range

$$10^{-6}\ \text{eV} < M < 10^{-3}\ \text{eV}\,. \tag{2}$$

The Lagrangian originally proposed for quantum chromodynamics (QCD) was of the simple form, analogous to quantum electrodynamics (QED),

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} - \frac{1}{2}\sum_{i}\bar{q}_{i,a}\gamma^{\mu}D^{ab}_{\mu}q_{i,b} \tag{3}$$

summed over the six quark flavors.

The simplicity of Eq. (3) was only temporary and became more complicated in 1975 by the discovery of instantons[7] which dictated[8,9] that an additional term in the QCD Lagrangian must be added

$$\Delta\mathcal{L}_{\text{QCD}} = \frac{\Theta}{64\pi^2}G^{\alpha}_{\mu\nu}\tilde{G}^{\mu\nu}_{\alpha}\,, \tag{4}$$

where $\tilde{G}_{\mu\nu}$ is the dual of $G_{\mu\nu}$. Although this extra term is an exact derivative, it cannot be discarded as a surface term because there is now a topologically nontrivial QCD vacuum with an infinite number of different values of the spacetime integral over Eq. (4) all of which correspond to $G^{\alpha}_{\mu\nu} = 0$. Normalized as in Eq. (4), the spacetime integral of this term must be an integer, and an instanton configuration changes this integer, or Pontryagin number, by unity.

When the quark masses are complex, an instanton changes not only Θ but also the phase of the quark mass matrix $\mathcal{M}_{\text{quark}}$ and the full phase to be considered is

$$\bar{\Theta} = \Theta + \arg\det\|\mathcal{M}_{\text{quark}}\|\,. \tag{5}$$

The additional term, Eq. (4), violates P and CP, and contributes to the neutron electric dipole moment whose upper limit[10] provides a constraint

$$\bar{\Theta} < 10^{-9} \tag{6}$$

which fine-tuning is the strong CP problem.

The hypothetical axion particle then arises from an ingenious technique to resolve Eq. (6), although as it turns out it may have been too ingenious. The technique is based on the Peccei–Quinn mechanism[11,12] which introduces a new global $\text{U}(1)_{\text{PQ}}$ symmetry which allows the vacuum to relax to $\bar{\Theta} = 0$. Because this $\text{U}(1)_{\text{PQ}}$ symmetry is spontaneously broken, it gives rise[13,14] to a light pseudoscalar axion with mass in the range $100\ \text{keV} < M < 1\ \text{MeV}$. An axion in this mass range was excluded experimentally but then the theory was modified to one with an invisible axion[15–18] where the $\text{U}(1)_{\text{PQ}}$ symmetry is broken at a much higher scale f_a and the coupling of the axion correspondingly suppressed. Nevertheless, clever experiments to detect such so-called invisible axions were proposed.[19]

Over 20 years ago, in 1992, three papers[20–22] independently pointed out a serious objection to the invisible axion. The point is that the invisible axion potential is so fine-tuned that adding gravitational couplings for weak gravitational fields at the dimension-five level requires tuning of a dimensionless coupling g to be at least as small as $g < 10^{-40}$, more extreme than the tuning of $\bar{\Theta}$ in Eq. (6) that one is trying to avoid.

Although a true statement, it is not a way out of this objection to say that we do not know the correct theory of quantum gravity because for weak gravitational fields, as is the case almost everywhere in the visible universe, one can use an effective field theory as discussed in Ref. 23. To our knowledge, this serious objection to the invisible axion which has been generally ignored since 1992 has not gone away and therefore the invisible axion may not exist. This issue is not settled but in a study of DM, we may *pro tempore* assume that there is no axion.

There would remain the strong CP problem of Eq. (6). One other solution would be a massless up-quark but this is disfavored by lattice calculations.[24] For the moment, Eq. (6) must be regarded as fine-tuning. We recall that the ratio of any neutrino mass to the top quark mass in the Standard Model satisfies

$$\left(\frac{M_\nu}{M_t}\right) < 10^{-12}\,. \tag{7}$$

2.2. *WIMPs*

WIMP is generally meant an unidentified elementary particle with mass in the range, say, between 10 GeV and 1000 GeV and with scattering cross-section with nucleons (N) satisfying, according to the latest unsuccessful WIMP direct searches,

$$\sigma_{\mathrm{WIMP}-N} < 10^{-44}\ \mathrm{cm}^2 \tag{8}$$

which is somewhat smaller than, but roughly comparable to, the characteristic strength of the known weak interaction.

The WIMP particle must be electrically neutral and be stable or have an extremely long lifetime. In model-building, the stability may be achieved by an *ad hoc* discrete symmetry, for example, a Z_2 symmetry under which all the Standard Model particles are even and others are odd. If the discrete symmetry is unbroken, the lightest odd state must be stable and therefore a candidate for a DM. In general, this appears contrived because the discrete symmetry is not otherwise motivated.

By far the most popular WIMP example came from weak scale supersymmetry where a discrete R symmetry has the value $R = +1$ for the Standard Model particles and $R = -1$ for all the sparticles. Such an R parity is less *ad hoc* being essential to prevent too-fast proton decay. The lightest $R = -1$ particle is stable and, if not a gravitino which has the problem of too-slow decay in the early universe, it was the neutralino,[25] a linear combination of zino, bino and higgsino. The neutralino provided an attractive candidate.

A problem for the neutralino is at the LHC where weak scale supersymmetry not many years ago confidently predicted sparticles (gluinos, etc.) at the weak scale ~ 250 GeV there is no sign of a gluino with mass anywhere up to at least 1700 GeV (Refs. 26–28) so weak scale supersymmmetry may not exist. Nevertheless, the jury is still out and numerous physicists remain optimistic. For present purposes, we can, again *pro tempore*, assume there is no WIMP.

It is worth recalling briefly the history of weak scale supersymmetry. The Standard Model[29–32] was in place by 1971 and its biggest theoretical problem was that, unlike QED with only log divergences, the scalar sector of the Standard Model generates quadratic divergences which destabilize the mass of the BEH boson.

When supersymmetric field theories were invented[33] in 1974, they provided an elegant solution of the quadratic divergence problem and hence immediately became popular. Even more so in 1983 when the neutralino was identified[25] as a DM candidate and more so again in 1991 when it was pointed out[34] that grand unification works better with the supersymmetric partners are included.

With the benefit of hindsight, these motivations for supersymmetry can all be otherwise realized. The quadratic divergence can cancel[35] in non-supersymmetric quiver theories. A DM candidate can be invented, in an *ad hoc* fashion, within conformality model building.[36] Historically, the neutralino appeared in particle phenomenology research *before* the WIMP acronym entered[37] the lexicon of cosmology. It is an important point that the WIMP idea came from weak scale supersymmetry.

Precise unification[34] with supersymmetry by adding one parameter, a common sparticle mass, was not miraculous but had at least a 20% probability as shown in Ref. 38. Other precise grand unifications[39] are known without supersymmetry in conformality model building. If we do need[b] as a replacement for weak scale supersymmetry, conformal invariance is a contender as discussed in 1998[40] and a number of subsequent papers as well more recently in Refs. 41 and 42.

Run 2 of the LHC is not necessarily doomed if WIMPs and sparticles do not exist. An important question, independent of naturalness but surely related to anomalies, is the understanding of why there are three families of quarks and leptons. For that reason, Run 2 may discover additional gauge bosons, siblings of the $W^{\pm}$ and Z^0, as discussed in Refs. 43 and 44.

The many attempts to detect WIMPs directly and indirectly are discussed in Ref. 5.

3. MACHOs

MACHOs are commonly defined[45] by the notion of compact objects used in astrophysics[46] as the end products of stellar evolution when most of the nuclear fuel has been expended. They are usually defined to include white dwarfs, neutron stars,

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[b] The unnaturalness exhibited in Eqs. (6) and (7) of the text both may suggest new physics beyond the Standard Model.

black holes, brown dwarfs and unassociated planets, all equally hard to detect because they do not emit any radiation.

This narrow definition implies, however, that MACHOs are composed of normal matter which is too restrictive in the special case of black holes. It is here posited that black holes of arbitrarily high mass up to 100,000 $M_\odot$ can be produced primordially as calculated and demonstrated in Ref. 47. Nevertheless, the acronym MACHO still nicely applies to DM IMBHs which are massive, compact and in the halo.

Unlike the axion and WIMP elementary particles which would have a definite mass, the black holes will have a range of masses. The lightest Primordial Black Hole (PBH) which has survived for the age of the universe has a lower mass limit

$$M_{\mathrm{PBH}} > 10^{-18}\, M_\odot \sim 10^{36}\ \mathrm{TeV} \tag{9}$$

already 36 orders of magnitude heavier than the heaviest would-be WIMP. This lower limit comes from the lifetime formula derivable from Hawking radiation[48]

$$\tau_{\mathrm{BH}}(M_{\mathrm{BH}}) \sim \frac{G^2 M_{\mathrm{BH}}^3}{\hbar c^4} \sim 10^{64} \left(\frac{M_{\mathrm{BH}}}{M_\odot}\right)^3 \ \text{years}\,. \tag{10}$$

Because of observational constraints[49,50] the DM constituents must generally be another 20 orders of magnitude more massive than the lower limit in Eq. (9). We assert[51–53] that most DM black holes are in the mass range between 100 and 100,000 times the solar mass. The name IMBHs is appropriate because they lie in mass above stellar-mass black holes and below the supermassive black holes which reside in galactic cores.

The possibility that primordial black holes can be formed with intermediate and higher masses has been established by an existence theorem discussed in a related paper.[54]

Let us discuss three methods (there may be more) which could be used to search for DM IMBHs. While doing so, we shall clarify what limits, if any, can be deduced from present observational knowledge.

Before proceeding, it is appropriate first to mention the important Xu–Ostriker upper bound of about a million solar masses from galactic disk stability[55] for any MACHO residing inside the galaxy.

4. Wide Binaries

There exist in the Milky Way pairs of stars which are gravitationally bound binaries with a separation more than 0.1 pc. These wide binaries retain their original orbital parameters unless compelled to change them by gravitational influences, for example, due to nearby IMBHs.

Because of their very low binding energy, wide binaries are particularly sensitive to gravitational perturbations and can be used to place an upper limit on, or to detect, IMBHs. The history of employing this ingenious technique is regretfully checkered. In 2004, a fatally strong constraint was claimed by an Ohio State University group[6] in a paper entitled "End of the MACHO Era" so that, for researchers who have time to read only titles and abstracts, stellar and higher mass constituents of DM appeared to be totally excluded.

Five years later in 2009, however, another group this time from Cambridge University[56] re-analyzed the available data on wide binaries and reached a quite different conclusion. They questioned whether *any* rigorous constraint on MACHOs could yet be claimed, especially as one of the important binaries in the earlier sample had been mis-identified.

Because of its checkered history, it seems wisest to proceed with caution but to recognize that wide binaries represent a potentially useful source both of constraints on, and the possible discovery of, DM IMBHs.

A further study of wide binaries[57] attempted to place limits on MACHOs. However, unlike microlensing which has positive signals, wide binary analysis is a null experiment and we remain skeptical of any limits claimed.

5. Distortion of the CMB

This approach hinges on the phenomenon of accretion of gas onto the IMBHs. The X-rays emitted by such accretion of gas are downgraded in frequency by cosmic expansion and by Thomson scattering becoming microwaves which distort the CMB, both with regard to its spectrum and to its anisotropy.

One impressive calculation of this effect[50] employs a specific model for the accretion, the Bondi–Hoyle model,[58] and carries through the computation all the way up to a point of comparison with data from FIRAS on CMB spectral distortions,[59] where FIRAS was a sensitive device attached to the COBE satellite. Unfortunately, the paper includes the limits from wide binaries discussed in Ref. 6, *ut supra*, and preceded the corrective paper,[56] so its results might have been influenced.

The results obtained from this approach are interesting if one can be certain that the gas accretion, subsequent X-ray emission and downgrading are well modeled. Like wide binaries, CMB distortion is indirect but could in future lead to useful bounds on, or the possible discovery of, DM IMBHs.

6. Microlensing

Microlensing is the most direct experimental method and has the big advantage that it has successfully found examples of MACHOs. The MACHO Collaboration used a method which had been proposed[c] by Paczynski[60] where the amplification of a distant source by an intermediate gravitational lens is observed. The MACHO Collaboration discovered several striking microlensing events whose light curves are exhibited in Ref. 49. The method certainly worked well for $M < 100\, M_\odot$ and

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[c] We have read that such gravitational lensing was later found to have been calculated in unpublished 1912 notes by Einstein who did not publish perhaps because at that time he considered its experimental measurement impracticable.

so should work equally well for $M > 100\, M_\odot$ provided one can devise a suitable algorithm and computer program to scan enough sources.

The duration of a given lensing event is proportional to the square root of the lensing mass and numerically is given by ($\hat{t}$ is longevity)

$$\hat{t} \simeq 0.2 \text{ yr} \left(\frac{M_{\text{lens}}}{M_\odot} \right)^{1/2} , \tag{11}$$

where a transit velocity 200 km/s is assumed for the lensing object.

The MACHO Collaboration investigated lensing events with durations ranging between about two hours and two years. From Eq. (11) this corresponds to MACHO masses between approximately $10^{-6}\, M_\odot$ and $100\, M_\odot$.

The total number and masses of objects discovered by the MACHO Collaboration could not account for all the DM known to exist in the Milky Way. At most 10% could be explained. To our knowledge, the experiment ran out of money and was essentially abandoned in the year 2000. But perhaps the MACHO Collaboration and its funding agency were too easily discouraged.

What is being suggested is that the other 90% of the DM in the Milky Way is in the form of MACHOs which are more massive than those detected by the MACHO Collaboration, and which almost certainly could be detected by a straightforward extension of their techniques. In particular, the expected microlensing events have a duration ranging up to two centuries. Let us consider the entries of Table 1 which merit discussion both with respect to the proposed microlensing experiment and briefly with respect to the entropy of the universe.

We simplify the visible universe without losing anything important by regarding it as containing exactly 10^{11} galaxies, each with mass (dominantly DM) of exactly $10^{12}\, M_\odot$. The first three columns of Table 1 consider one halo of DM. To a first approximation, we can temporarily ignore the normal matter. The fourth column gives the additive entropy of the universe for well separated halos and the fifth column gives the corresponding microlensing event duration in years.

For a black hole with mass $M_{\text{BH}} = \eta M_\odot$, the dimensionless entropy is $S_{\text{BH}}/k \sim 10^{77}\eta^2$, in other words

$$S_{\text{BH}}/k = 10^{77} \left(\frac{M_{\text{BH}}}{M_\odot} \right)^2 . \tag{12}$$

If we study the first five rows of Table 1, we note that for a given total halo mass, $M_{\text{Halo}} = 10^{12}\, M_\odot$, a smaller number of heavier black holes gives higher entropy because $S_{\text{BH}} \propto M_{\text{BH}}^2$. Within galactic halos, the black hole masses are restricted to be below $10^6\, M_\odot$ by the disk stability already mentioned. Various arrangements of the allowed black hole mass function were explored in Ref. 61. Arguments using the concept of the entropy of the universe, together with the second law of thermodynamics, are strongly suggestive of many more black holes than the stellar and supermassive types already identified for the simple reason that black holes are, by far, the most efficient concentrators of entropy.

Table 1. Microlensing Duration ($\hat{t}$) for the case of n IMBHs per halo. IMBH mass $= \eta M_\odot$. Halo mass $= 10^{12}\, M_\odot$. Universe mass $= 10^{23}\, M_\odot$. See also Ref. 62.

n/Halo $\log_{10} n$	$M = \eta M_\odot$ $\log_{10} \eta$	Halo entropy $\log_{10}(S_{\mathrm{Halo}}/k)$	Universe entropy $\log_{10}(S_{\mathrm{Universe}}/k)$	Duration $\hat{t}$ (years)
10	2	91	102	2
9	3	92	103	6
8	4	93	104	20
7	5	94	105	60
6	6	95	106	200
0	*12*	*101*	*112*	n/a

The sixth and last row in Table 1 illustrates how if a halo hypothetically collapsed into one large black hole, its entropy would be $S^{\mathrm{max}}_{\mathrm{Halo}}/k \sim 10^{101}$. If the superluminal accelerated expansion prevents coalescence of such collapsed halos the additive entropy of the universe's interior would be $S_{\mathrm{Universe}}/k \sim 10^{112}$. If, hypothetically, all the halos would instead combine to one very large black hole with mass $10^{23}\, M_\odot$, the entropy would be $S_{\mathrm{Universe}}/k \sim 10^{123}$. The Schwarzschild radius of this very large black hole is $R = 10^{23} \times 3$ km ~ 30 Gly, not far below the comoving radius (~ 45 Gly) of the visible universe.

The discussion of the previous paragraph implies, as has been discussed elsewhere, that the visible universe is, in some sense, close to itself being a black hole inside of which we live. This curious fact seems to have no bearing on DM but may be relevant to the more difficult problem of dark energy.

Microlensing experiments involve systematic scans of millions of distant star sources because it requires accurate alignment of the star and the intermediate lensing MACHO. Because the experiments are already highly computer intensive, it makes us more optimistic that the higher duration events can be successfully analyzed. Study of an event lasting two centuries should not necessitate that long an amount of observation time. It does require suitably ingenious computer programming to track light curves and distinguish them from other variable sources. This experiment is undoubtedly extremely challenging, but there seems no obvious reason it is impracticable.

A fraction of the resources currently being thrown at WIMP searches could be enough to support this desirable pursuit of high-duration microlensing observations.

7. Discussion

Axions may not exist for theoretical reasons[20–22] discovered in 1992. Weak scale supersymmetry may not exist for the experimental reason[26–28] of its non-discovery at the LHC. The idea that DM experiences weak interactions (WIMPs) came historically from the appearance of an appealing DM constituent, the neutralino, in the theory of weak scale supersymmetry.

The only interaction which we know for certain to be experienced by DM is gravity and the simplest assumption is that gravity is the only force coupled to DM. Why should the DM experience the weak interaction when it does not experience the strong and electromagnetic interactions? If it does not, then terrestrial experiments searching for DM by either direct detection or production would be doomed to failure.

We began with four candidates for DM constituent: (1) axion, (2) WIMP, (3) brown dwarf, understood to include all compact baryonic objects and (4) black hole. We *pro tempore* eliminated the first two by hopefully persuasive arguments, made within the context of an overview of particle phenomenology. We eliminated the third by the upper limit on baryons imposed by robust Big Bang Nucleosynthesis (BBN) calculations.

We should remain open to the possibility that axions and/or WIMPs may exist and that the DM might involve them as well, with PIMBHs giving only the dominant contribution. We also point out that black holes as constituents of DM in a general sense have been previously discussed e.g. in Ref. 63 for many years.

We assert that PIMBHs can constitute almost all DM while maintaining consistency with the BBN calculations. This is an important point because distinguished astronomers have written an opposite assertion e.g. Begelman and Rees[64] state, on their page 256, that black holes cannot form more than 20% of DM because the remainder is non-baryonic and Freese[5] states similarly, on her page 99, that, because of the constraint imposed by BBN, astrophysical black holes cannot provide the DM in galaxies.

Both sources are making an implicit assumption which does not apply to the PIMBHs which we assert comprise almost all DM. That assumption is that black holes can be formed only as the result of the gravitational collapse of baryonic stars. We are claiming, on the contrary, that DM black holes can be, and the majority must be, formed primordially in the early universe as calculated and demonstrated in Ref. 47, and endorsed by Ref. 65, unconstrained by the BBN upper limit on baryons. Earlier works on PBHs, and DM candidates, include Refs. 66–69.

Our proposal is that the Milky Way contains between 10 million and 10 billion massive black holes each with between 100 and 100,000 times the solar mass. Assuming the halo is a sphere of radius a 100,000 light years the typical separation is between 100 and 1000 light years which is also the most probable distance of the nearest PIMBH to the Earth. At first sight, it may be surprising that such a number of massive black holes could have remained undetected in the Milky Way. On second thoughts, it appears reasonable when one bears in mind their large mean separation of 100 to 1000 light years and their relatively small size, all being physically smaller than the Sun.

Of the detection methods discussed, extended microlensing observations seem the most promising and writing the present paper will have been worthwhile if efforts to detect higher duration microlensing events are hereby encouraged. It will be exceptionally rewarding if most of the DM in our galaxy is confirmed, by microlensing techniques or otherwise, to be in the form of IMBHs.

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References

1. Planck Collab., arXiv:1502.01589.
2. D. W. Sciama, *Modern Cosmology and the Dark Matter* (Cambridge Univ. Press, 2008).
3. R. H. Sanders, *The Dark Matter Problem*: *A Historical Perspective* (Cambridge Univ. Press, 2014).
4. (Ed.) G. Bertone, *Particle Dark Matter*, *Observations*, *Models and Searches* (Cambridge Univ. Press, 2013).
5. K. Freese, *The Cosmic Cocktail*: *Three Parts Dark Matter* (Princeton Univ. Press, 2014).
6. J. Yoo, J. Chaname and A. Gould, *Astrophys. J.* **601**, 311 (2004).
7. A. A. Belavin, A. M. Polyakov, A. S. Schwartz and Yu. S. Tyupkin, *Phys. Lett. B* **59**, 85 (1975).
8. G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976).
9. G. 't Hooft, *Phys. Rev. D* **14**, 3432 (1976).
10. C. A. Baker, D. D. Doyle, P. Geltenbort, K. Green, M. G. D. van der Grinten, P. G. Harris, P. Laydjiev, S. N. Ivanov, D. J. R. May, J. M. Pendleburg, J. D. Richardson, D. Shiers and K. F. Smith, *Phys. Rev. Lett.* **97**, 131801 (2006).
11. R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).
12. R. D. Peccei and H. R. Quinn, *Phys. Rev. D* **16**, 1791 (1977).
13. S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978).
14. F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978).
15. M. Dine, W. Fischler and M. Srednicki, *Phys. Lett. B* **104**, 199 (1981).
16. J. E. Kim, *Phys. Rev. Lett.* **43**, 103 (1979).
17. A. Zhitnitsky, *Sov. J. Nucl. Phys.* **31**, 260 (1980).
18. M. Shifman, A. Vainshtein and V. Zakharov, *Nucl. Phys. B* **166**, 493 (1980).
19. P. Sikivie, *Phys. Rev. Lett.* **51**, 1415 (1983).
20. R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins and L. M. Widrow, *Phys. Lett. B* **282**, 132 (1992).
21. M. Kamionkowski and J. March-Russell, *Phys. Lett. B* **282**, 137 (1992).
22. S. M. Barr and D. Seckel, *Phys. Rev. D* **46**, 539 (1992).
23. J. F. Donoghue, *Phys. Rev. D* **50**, 3874 (1994).
24. D. R. Nelson, G. T. Fleming and G. W. Kilcup, *Phys. Rev. Lett.* **90**, 021601 (2003).
25. H. Goldberg, *Phys. Rev. Lett.* **50**, 1419 (1983).
26. ATLAS Collab., *JHEP* **10**, 054 (2015).
27. CMS Collab., *Phys. Rev. D* **91**, 052018 (2015).
28. CMS Collab., *Phys. Lett. B* **744**, 184 (2015).
29. S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961).
30. S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).
31. S. L. Glashow, J. Iliopoulos and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
32. G. 't Hooft, *Nucl. Phys. B* **35**, 167 (1971).
33. J. Wess and B. Zumino, *Nucl. Phys. B* **70**, 39 (1974).

34. U. Amaldi, W. de Boer and H. Furstenau, *Phys. Lett. B* **260**, 447 (1991).
35. X. Calmet, P. H. Frampton and R. M. Rohm, *Phys. Rev. D* **72**, 055003 (2005).
36. P. H. Frampton, *Mod. Phys. Lett. A* **22**, 931 (2007).
37. E. W. Kolb and M. S. Turner, *The Early Universe*, Frontiers in Physics (Addison-Wesley, 1990).
38. U. Amaldi, W. De Boer, P. H. Frampton, H. Furstenau and J. T. Liu, *Phys. Lett. B* **281**, 374 (1992).
39. P. H. Frampton, *Mod. Phys. Lett. A* **18**, 1377 (2003).
40. P. H. Frampton, *Phys. Rev. D* **60**, 041901 (1999).
41. G. 't Hooft, arXiv:1410.6675.
42. P. D. Mannheim, arXiv:1506.01399.
43. P. H. Frampton, *Phys. Rev. Lett.* **69**, 2889 (1992).
44. P. H. Frampton, *Phys. Lett. B* **747**, 187 (2015).
45. K. Griest, *Astrophys. J.* **366**, 412 (1991).
46. S. L. Shapiro and S. A. Teukolsky, *Black Holes*, *White Dwarfs and Neutron Stars*: *The Physics of Compact Objects* (John Wiley & Sons, 1983).
47. P. H. Frampton, M. Kawasaki, F. Takahashi and T. T. Yanagida, *J. Cosmol. Astropart. Phys.* **1004**, 023 (2010).
48. S. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
49. MACHO Collab. (C. Alcock *et al.*), *Astrophys. J.* **542**, 281 (2000).
50. M. Ricotti, J. P. Ostriker and K. J. Mack, *Astrophys. J.* **680**, 829 (2008).
51. P. H. Frampton, *J. Cosmol. Astropart. Phys.* **0910**, 016 (2009).
52. P. H. Frampton, *Nucl. Phys. B*: *Proc. Suppl.* **200–202**, 176 (2010).
53. P. H. Frampton, *AIP Conf. Proc.* **1232**, 53 (2010).
54. P. H. Frampton, *Mod. Phys. Lett. A* **31**, 1650064 (2016).
55. G. H. Xu and J. P. Ostriker, *Astrophys. J.* **437**, 184 (1994).
56. D. P. Quinn, M. I. Wilkinson, M. J. Irwin, J. Marshall, A. Koch and V. Belokurov, *Mon. Not. R. Astron. Soc.* **396**, 11 (2009).
57. M. A. Monroy-Rodriguez and C. Allen, arXiv:1406.5169.
58. H. Bondi and F. Hoyle, *Mon. Not. R. Astron. Soc.* **104**, 273 (1944).
59. D. J. Fixsen, E. S. Cheng, J. M. Gales, J. C. Mather, R. A. Shafer and E. L. Wright, *Astrophys. J.* **473**, 576 (1996).
60. B. Paczynski, *Astrophys. J.* **304**, 1 (1986).
61. P. H. Frampton and K. J. Ludwick, *Astropart. Phys.* **34**, 617 (2011).
62. P. H. Frampton, arXiv:0806.1707.
63. J. Garcia-Bellido, A. Linde and D. Wands, *Phys. Rev. D* **54**, 6040 (1996).
64. M. Begelman and M. Rees, *Gravity's Fatal Attraction*: *Black Holes in the Universe*, 2nd edn. (Cambridge Univ. Press, 2010).
65. B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *Phys. Rev. D* **81**, 104019 (2010).
66. K. Kohri, T. Nakama and T. Suyama, *Phys. Rev. D* **90**, 083514 (2014).
67. K. M. Belotsky, A. D. Dmitriev, E. A. Esipova, V. A. Gani, A. V. Grobov, M. Yu. Khlopov, A. A. Kirillov, S. G. Rubin and I. V. Svadkovsky, *Mod. Phys. Lett. A* **29**, 1440005 (2014).
68. H. Terazawa, *J. Phys. Soc. Jpn.* **58**, 3555 (1989).
69. E. Bugaev and P. Klimai, *Phys. Rev. D* **83**, 083521 (2011).