Chiral Dilepton Model and the Flavor Question

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A chiral model based on a gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ contains dilepton gauge bosons and new quarks with exotic charges $-\frac{4}{3}$ and $+\frac{5}{3}$. Although coincident at low energy with the standard model, in the extended theory the third quark family is treated differently from the first two, and anomaly cancellation requires that the number of families be equal to the number of quark colors.

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In the present Letter we suggest that some aspects of the standard model [1] might be understood by embedding the three-family version in a group $SU(3)_C \times SU(3)_L \times U(1)_X$ (hereafter 3-3-1 model) with a corresponding enlargement of the quark representations. In particular, the number of families will be related by anomaly cancellation to the number of quark colors, and the third family including the top quark will be treated differently from the first and second families; in the $SU(3)_C \times SU(2)_L \times U(1)_Y$ low-energy limit all three families appear similarly and cancel anomalies separately.

The present model arose from the need to find a chiral theory of dilepton gauge bosons which have recently been studied in detail on the basis of SU(15) unified theory [2]. The latter has the somewhat unpleasant feature of being a nonchiral theory with anomalies canceled artificially by mirror fermions; here we present a dilepton theory which is chiral and has nontrivial anomaly cancellation.

The 3-3-1 model treats the leptons democratically in each of the three families. These color singlets are in antitriplets of $SU(3)_L$:

$$3_L^* = \begin{pmatrix} e^- \\ \nu_e \\ e^+ \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \\ \mu^+ \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \\ \tau^+ \end{pmatrix}, \tag{1}$$

each with $X=0$ since the electric charge operator is embedded as $Q=\frac{1}{2}\lambda_L^3+(\sqrt{3}/2)\lambda_L^8+X\frac{3}{2}^{1/2}\lambda^9$ or $Q=T_3+\frac{1}{2}Y$ with $Y=\sqrt{3}\lambda_L^8+\sqrt{6}X\lambda^9$. Here the normalization is $\mathrm{Tr}(\lambda_L^a\lambda_L^b)=2\delta^{ab}$ and $\lambda^9=\frac{2}{3}^{1/2}\mathrm{diag}\,(1,1,1)$.

The quarks in the first family are color triplets and transform as left-handed triplets plus three singlets under $SU(3)_L$:

$$3_L = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}_L, \quad 1_L=(\bar{u}_a)_L,\ (\bar{d}_a)_L,\ (\bar{D}_a)_L\,. \tag{2}$$

The 3_L triplet has $X=-\frac{1}{3}$ and the singlets carry $X=-\frac{2}{3}$, $+\frac{1}{3}$, and $+\frac{4}{3}$, respectively. The new quark D has electric charge $Q=-\frac{4}{3}$. Here $\alpha=1,2,3$ for color $SU(3)_C$.

In the second family the quarks (c^a,s^a,S^a) with a new S quark having $Q=-\frac{4}{3}$ are arranged similarly to the first family in (2) above.

As already noted, the quarks of the third family are treated differently. The color triplet quarks are in a left-handed *antitriplet* and three singlets under $SU(3)_L$:

$$3_L^* = \begin{pmatrix} b^a \\ t^a \\ T^a \end{pmatrix}, \quad 1_L=(\bar{b}_a)_L,\ (\bar{t}_a)_L,\ (\bar{T}_a)_L\,. \tag{3}$$

This antitriplet has $X=+\frac{2}{3}$ and the singlets carry $X=+\frac{1}{3}$, $-\frac{2}{3}$, and $-\frac{5}{3}$, respectively. The new quark T has electric charge $Q=+\frac{5}{3}$.

Before discussing the symmetry breaking of $SU(3)_L \times U(1)_X$ to $SU(2)_L \times U(1)_Y$ and the resulting mass spectrum we shall explain the nontrivial anomaly cancellation of this model. There are five triangle anomalies which are potentially troublesome; in a self-explanatory notation these are $(3_C)^3$, $(3_C)^2X$, $(3_L)^3$, $(3_L)^2X$, and X^3. The QCD anomaly $(3_C)^3$ is absent because QCD is, as usual, vectorlike. $(3_C)^2X$ vanishes because the quarks are in nine color triplets with net $X=0$ and nine antitriplets also with net $X=0$. The pure $SU(3)_L$ anomaly vanishes because there is an equal number of 3_L and 3_L^*. $(3_L)^2X$ cancels because the leptons have $X=0$ and the quarks are in six triplets 3_L with $X=-\frac{1}{3}$ and three antitriplets 3_L^* with $X=+\frac{2}{3}$. Finally the X^3 cancellation can be checked by a little algebra: The three quark families contribute, respectively, $+6+6-12=0$.

It is especially interesting that this anomaly cancellation takes place *between* families. Each individual family possesses nonvanishing $(3_L)^3$, $(3_L)^2X$, and X^3 anomalies. Only with a matching of the number of families with the number of quark colors does the overall anomaly vanish. This provides a first step towards answering the flavor question, often cited as Rabi's famous remark about the muon: "Who ordered that?"

The breaking of the 3-3-1 model to the standard model is achieved by a vacuum expectation value (VEV) of an $X=+1$ triplet $\langle\Phi^a\rangle=U\delta^{a3}$. This gives mass $\Lambda_{D,S,T}U$ to the new quarks D,S,T, where Λ_i are the Yukawa couplings. It also provides mass to five gauge bosons: the dileptons $(Y^{\pm\pm},Y^{\pm})$ and Z'. Dileptons receive a mass squared $(g^2/2)U^2$. The W^8-X mass matrix

$$\begin{pmatrix} \frac{1}{3}g^2U^2 & -(\sqrt{2}/3)gg_xU^2 \\ -(\sqrt{2}/3)gg_xU^2 & \frac{2}{3}g_x^2U^2 \end{pmatrix} \tag{4}$$

is easily diagonalized to identify

$$Z'_\mu = \frac{1}{(g^2+2g_x^2)^{1/2}}[gW_\mu^8+\sqrt{2}g_xX_\mu]\,, \tag{5}$$

$$B_\mu = \frac{1}{(g^2+2g_x^2)^{1/2}}[\sqrt{2}g_xW_\mu^8-gX_\mu]\,, \tag{6}$$

as expected from the fact that $Y=\sqrt{3}\lambda^8+\sqrt{6}X\lambda^9$ with $\lambda^9=\frac{2}{3}{}^{1/2}\mathrm{diag}(1,1,1)$. Thus Z' has a mass squared $\frac{1}{3}(g^2+2g_x^2)U^2$.

If we assume a normalization prompted by potential unification in a more complete theory [3]

$$\tfrac{1}{4}g_x^2\mathrm{Tr}X^2=g^2\mathrm{Tr}(T_3)^2\,, \tag{7}$$

we find for the given fermion representations of (2) and (3) above that $g_x^2=4g^2/5$ and hence

$$M(Y)=\sqrt{15/26}M(Z')\,. \tag{8}$$

Later we shall derive a conservative lower bound $M(Z') > 300$ GeV so that Eq. (8) implies a dilepton mass $M(Y)\geq 230$ GeV nicely consistent with experimental bounds [4] and making the dilepton gauge bosons possibly accessible at the high-energy colliders Large Electron Positron–Large Hadron Collider, Superconducting Super Collider, Large Hadron Collider, and Next Linear Collider.

Electroweak breaking is achieved by VEVs of two triplets $\langle\phi^\alpha\rangle=v\delta^{\alpha 2}$ (with $X=0$) and $\langle\phi'^\alpha\rangle=v'\delta^{\alpha 1}$ (with $X=-1$), and a doublet VEV of a sextet ($X=0$) $\langle H^{\alpha\beta}\rangle = y\sqrt{10}(\delta^{\alpha 1}\delta^{\beta 3}+\delta^{\alpha 3}\delta^{\beta 1})$. The dilepton masses now become $M^2(Y^{++})=(g^2/2)(U^2+v'^2)$ and $M^2(Y^+)=(g^2/2)(U^2+v^2+y^2)$. If $(v')^2=v^2+y^2$ a custodial global SU(2) is preserved which keeps $M^2(Y^+)=M^2(Y^{++})$.

The W^+ aquires mass $M^2(W^+)=(g^2/2)(v^2+v'^2+y^2)$ and does *not* mix with Y^+ because all VEVs respect lepton member $L=L_e+L_\mu+L_e$, while W^+ and Y^+ have differing lepton numbers. Note that we are imposing (except for a brief digression below) global L conservation in the Higgs potential, and that the exotic quarks D and S have $L=+2$ while T has $L=-2$.

The nonexotic quarks acquire masses through the Yukawa couplings to the ϕ and ϕ' scalars and their complex conjugates when ϕ and ϕ' obtain VEVs.

The charged leptons acquire an antisymmetric (in flavor space) mass matrix from ϕ and a symmetric one from H, thus allowing arbitrary masses for e^-, μ^-, τ^-; this is the reason the sextet VEV is necessary, since otherwise $m(\mu)=m(\tau)$ is an unacceptable consequence. The three neutrinos remain exactly massless both because L is still respected and because right-handed neutrinos have been omitted as in the minimal standard model.

The nongluonic neutral gauge bosons W_3-W_8-X have a mass matrix

$$\begin{pmatrix} \frac{g^2}{4}(v^2+v'^2+y^2) & \frac{g^2}{4\sqrt{3}}(-v^2+v'^2-y^2) & -\frac{gg_x}{\sqrt{6}}v'^2 \\ \frac{g^2}{4\sqrt{3}}(-v^2+v'^2-y^2) & \frac{g^2}{3}U^2+\frac{g^2}{12}(v^2+v'^2+y^2) & -\frac{gg_x}{3\sqrt{2}}(2U^2+v'^2) \\ -\frac{gg_x}{\sqrt{6}}v'^2 & -\frac{gg_x}{3\sqrt{2}}(2U^2+v'^2) & \frac{2}{3}g_x^2(U^2+v'^2) \end{pmatrix}. \tag{9}$$

By diagonalizing this mass matrix we can identify the three physical states γ, Z, and Z' and estimate how the Z' perturbs [5] the standard model predictions. The photon γ is massless, and the tight empirical limits on $M^2(Z)$, $M^2(W^\pm)$, and the electroweak mixing angle lead to a lower bound on $M^2(Z')$ and hence, though Eq. (8), on the dilepton mass $M^2(Y)$. It is convenient to parametrize the corrections in terms of $\xi=[M(Z)^2/M(Z')^2]_0$, where the subscript denotes lowest order. It is sufficient to set $v'=y=0$ in (9) and use Eqs. (5),(6) to postmultiply by

$$\frac{1}{N}\begin{pmatrix} N & 0 & 0 \\ 0 & \sqrt{2}g_x & -g \\ 0 & +g & \sqrt{2}g_x \end{pmatrix} \tag{10}$$

and premultiply by its transpose; here $N=(g^2+2g_x^2)^{1/2}$. This gives a partially diagonalized matrix in the W_3-B-Z' basis. Now postmultiply and premultiply by the idempotent matrix

$$\begin{pmatrix} \sin\theta & \cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

to obtain in the $A-Z-Z'$ basis the symmetric mass matrix

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{L^2}{12N^2}g^2v^2 & \frac{L}{12N^2}g^3v^2 \\ 0 & \frac{L}{12N^2}g^3v^2 & \frac{1}{3}N^2U^2+\frac{1}{12N^2}g^4v^2 \end{pmatrix}, \tag{12}$$

where we defined the useful variable $L^2=3g^2+8g_X^2$. The mixing angle is $\sin^2\theta=2g_X^2/L^2$ ($=\frac{8}{47}\approx 0.17$ in a hypothetical limit [3]). We then find that

$$\xi=g^2v^2L^2/4N^4U^2. \tag{13}$$

It is interesting that the formula $\sin^2\theta=2g_X^2/L$ requires $\sin^2\theta$ to be below $\frac{1}{4}$ at the new symmetry-breaking scale. In the standard model, $\sin^2\theta(\mu)=0.233$ at $\mu=M(Z)$ becomes 0.246 at $\mu=1$ TeV and 0.250 at $\mu=2.2$ TeV; assuming g_X^2 is not too much larger than g^2 (despite the arbitrariness in normalization of X) suggests upper bounds

$$M(Z')<1000\ \mathrm{GeV}, \tag{14}$$

$$M(Y)<760\ \mathrm{GeV}. \tag{15}$$

From (12) one can deduce that the $\Delta M(Z)^2$ mass shift is of order

$$M(Z)^2=M(Z)_0^2[1-(1-4\sin^2\theta)\xi/3+O(\xi^2)], \tag{16}$$

while $M(W)^2$ is unshifted at leading order. This means that the experimental input on $M(Z)$, $M(W)$, $\sin^2\theta$ from LEP suggest very conservatively only the weak lower bounds

$$M(Z')>300\ \mathrm{GeV}, \tag{17}$$

$$M(Y)>230\ \mathrm{GeV}, \tag{18}$$

as mentioned earlier. It seems possible that these lower bounds may be refined by a more critical comparison with these experimental data and/or by detailed consideration of flavor-changing neutral currents.

As one generalization of the minimal model we briefly consider giving a small VEV to the $SU(2)_L$ triplet component $\langle H^{22}\rangle\neq 0$. This breaks lepton number, mixes $Y^\pm$ and $W^\pm$, and gives a symmetric mass matrix to the neutrinos. But it leads to a triplet Majoron [6] which seems to be excluded experimentally.

In conclusion, the features of the 3-3-1 model which make it interesting are (i) the more economic accommodation of the dileptons than provided by an SU(15) grand unified theory [2], (ii) the asymmetric treatment of the third quark family which may be related to the heaviness [7] of the top quark, and (iii) the fact that anomaly cancellation requires exactly three families, and provides a possible answer to the flavor question. An upper limit on the symmetry-breaking scale (U) can be placed by the requirement that $\sin^2\theta<\frac{1}{4}$, implying that the physics associated with the dilepton gauge bosons, the Z', and the D,S,T quarks will be accessible to the next generation of colliders.

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[3] Note that in the present presumably incomplete model, the couplings α_{3C} and α_{3L} do not unify at high energy since the corresponding renormalization-group beta functions satisfy $\beta_{3C}>\beta_{3L}$.

[4] See Carlson and Frampton (Ref. [2]).

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